Ottawa-Carleton Institute of Mathematics and Statistics

Dunton Tower 707 Telephone: 788-2152 Fax: 788-3536

The Institute

Director of the Institute: Vlastimil Dlab Associate Director: Erhard Neher

Students who wish to pursue studies in pure mathematics, applied mathematics, probability and statistics at the graduate level leading to a M.Sc. or a Ph.D. degree can do so in a joint program offered by the Department of Mathematics and Statistics at Carleton University and the Department of Mathematics at the University of Ottawa under the auspices of the Institute for Graduate Studies and Research in Mathematics and Statistics. The Institute is responsible for supervising the programs, regulations and student admissions, as well as providing a framework for interaction between the two departments at the research level.

The list below of all members of the Institute along with their research interests can be used as a guide to possible supervisors.

In addition to the programs administered by the Institute, the Department of Mathematics and Statistics at Carleton University offers several other programs.

In cooperation with the Department of Systems and Computer Engineering and the School of Computer Science at Carleton University, students may pursue a program leading to an M.Sc. in Information and Systems Science. For information see page 200.

In cooperation with the School of Computer Science and the Department of Systems and Computer Engineering at Carleton University and the Department of Computer Science at the University of Ottawa, students may pursue a program leading to a Master of Computer Science (M.C.S.); for information see page 181.

The Department of Mathematics and Statistics also offers a cooperative master's program in statistics in collaboration with the federal government, emphasizing practical training through work experience, along with sound training in statistical inference and basic probability theory. Requests for information and completed applications should be sent to the Director or Associate Director of the Institute.

Members of the Institute

N.U. Ahmed, Nonlinear Functional Analysis, Control Theory Mayer Alvo, Nonparametric Statistics, Sequential Analysis Amitava Bose, Stochastic Modelling, Probability Theory W.D. Burgess, Algebra, Non-Commutative Rings C.E. Castonguay, Demography Maurice Chacron, Division Algebras With Involution M.P. Closs, Native American Mathematics E.L. Cohen, Diophantine Equations Miklòs Csörgö, Probability and Statistics A.R. Dabrowski, Invariance Principles, Weakly **Dependent Variables** Daniel Daigle, Algebraic Geometry, Commutative Algebra D.A. Dawson, Stochastic Processes and Probability Theory J.D. Dixon, Group Theory, Algebra Computation Vlastimil Dlab, Finite Dimensional Algebras, Representation Theory Zhicheng Gao, Graph Theory C.K. Fong, Operator Theory C.W.L. Garner, Foundations of Geometry Thierry Giordano, Operator Algebras, Ergodic Theory J.E. Graham, Sampling Theory, Multivariate Analysis D.E. Handelman, K-theory, Operator Algebras, **Ring Theory** Kenneth Hardy, Computational Number Theory R.M. Herz-Fischler, History and Sociology of Mathematics B.G. Ivanoff, Probability, Point Processes, Martingales Barry Jessup, Rational Homotopy Daniel Krewski, Applied Statistics in Medicine E.O. Kreyszig, Partial Differential Equations, Numerical Analysis L.E. May, Numerical Analysis D.R. McDonald, Applied Probability Paul Mandl, Non-linear Partial Differential Equations Sam Melkonian, Non-linear Differential Equations S.E. Mills, Applied Statistics, Statistical Methods, Inference A.B. Mingarelli, Ordinary Differential Equations, **Difference** Equations

204 Joint Program for Mathematics and Statistics

M.J. Moore, Coding Theory B.C. Mortimer, Group Theory, Coding Theory Erhard Neher, Jordan Algebras, Algebraic Groups L.D. Nel, Nonnormable Analysis and Calculus J.N. Pandey, Generalized Functions, Partial Differential Equations J.C. Poland, *Group Theory* I.S. Pressman, Optimization, Algebra B.M. Puttaswamaiah, Group Representations and **Applications** M.L. Racine, Jordan Algebras Mizanur Rahman, Special Functions J.N.K. Rao, Sample Surveys Theory and Methods Luis Ribes, Group Theory R.B. Richter, Graph Theory, Combinatorics Ivan Rival, Combinatorics, Algorithms Wulf Rossmann, Lie Groups Damien Roy, Transcendental Number Theory, Diophantine Approximations Bahman Saffari, Analytic Number Theory A.K.Md.E. Saleh, Order Statistics, Mathematical **Statistics** H.H. Schirmer, Algebraic Topology P.J. Scott, Logic, Category Theory Jun Shao, Statistical Inference, Resampling Methods R.R. Sitter, Surveys, Biostatistics, Resampling, Design, Quality Barbara Szyszkowicz, Statistics B.J. Tomiuk, Banach Algebras Remì Vaillancourt, Partial Differential Equations, Numerical Methods K.S. Williams, Number Theory B.B. Winter, Applied Probability, Nonparametric **Statistics**

Master of Science

Admission Requirements

The normal requirement for admission to the master's program is an honours bachelor's degree in mathematics, or the equivalent, with at least high honours standing. Applicants holding a general (pass) degree with at least high honours standing may be admitted to a qualifying-year program. Their subsequent admission to the regular master's program depends on their performance during the qualifying-year program and will be decided no later than one year after admission to the qualifyingyear program. Details are outlined in the general section of this calendar. Students with outstanding academic performance and research promise while in the M.Sc. program may be permitted to transfer to the Ph.D. program without completing the M.Sc. program.

Program Requirements

The two options for the M.Sc. program are:Eight one-term courses (or equivalent) and a thesis

• Ten one-term courses (or equivalent) The courses must be chosen from those at the graduate level except that a student may take up to two one-term approved undergraduate courses at the fourth-year level to satisfy these requirements. Not all these courses may be taken in the same field of mathematics; at least two must be in another field. All master's students are required to participate actively in a seminar or project under the guidance of his/her adviser. A maximum of two one-term courses taken outside of the Department of Mathematics and Statistics at Carleton University or the Department of Mathematics at the University of Ottawa may be allowed for credit.

Students who plan to specialize in probability and statistics are strongly advised that during their master's program they include, where possible, the courses 70.450, 70.551 in mathematical statistics; 70.452, 70.555 in applied statistics, and 70.451, 70.571 in probability, together with two further oneterm courses in the Department of Mathematics and Statistics. In addition, a graduate course in another field, such as biology, bio-statistics, economics, computer science, systems analysis, and stochastic modelling, is highly recommended.

Doctor of Philosophy

Admission Requirements

The normal requirement for admission to the Ph.D. program is a master's degree in mathematics, or the equivalent, with at least high honours standing. Details are outlined in the general section of this calendar.

Program Requirements

The course requirements, which are determined at the time of admission, include a minimum of six one-term graduate courses (or equivalent) and a suitable thesis. Not all of these courses may be taken in the same field of mathematics; at least two must be in another field.

All candidates must take a comprehensive examination, and satisfy a language requirement. The language requirement is determined by the candidate's advisory committee and normally requires the ability to read mathematical literature in a language considered useful for his/her research or career, and other than the candidate's principal language of study.

Students specializing in mathematics and probability undertake a comprehensive examination in the following areas:

- The candidate's general area of specialization at the Ph.D. level
- Examinations on two topics chosen from algebra, analysis, probability, topology, and statistics. (This choice excludes the student's specialty.)

Students specializing in statistics must write an examination in the following areas:

- Mathematical statistics which includes multivariate analysis
- An examination in probability, and
- An examination in either i) applied statistics, or ii) in analysis

In all cases, the examination must be completed successfully within twenty months of initial registration in the Ph.D. program in the case of full-time students and within thirty-eight months of initial registration in the case of part-time students.

All Ph.D. candidates are also required to undertake a final oral examination on the subject of their thesis.

Selection of Courses

The following undergraduate courses may, with the approval of the Department of Mathematics and Statistics, be selected by master's candidates in partial fulfilment of their degree requirements:

Mathematics and Statistics

Mainematics and Statistics		
70.401	Vector Calculus	•
70.403	Functional Analysis	В
70.407	Measure Theory	C
70.415	Rings and Modules	ic
70.416	Group Theory	a
70.417	Commutative Algebra	a
70.418	Homological Algebra and Category	e
	Theory	ti
70.425	Introduction to General Topology	u
70.426	Introduction to Algebraic Topology	•
70.427	Foundations of Geometry	I
70.428	Introduction to Differentiable Manifolds	A
70.435	Analytic Number Theory	S
70.436	Algebraic Number Theory	F
70.445	Analytical Dynamics	L
70.446	Hydrodynamics and Elasticity	E
70.447	Tensor Analysis and Relativity Theory	р
70.450	Parametric Estimation	P
70.451	Probability Theory	
70.452	Sampling: Theory and Methods	*
70.453	Applied Multivariate Analysis	ŵ
70.456	Non-Parametric Methods	Ť
70.457	Statistical Inference	co
70 450	Ctashart's Madala	et

70.458 Stochastic Models

- 70.459 Stochastic Optimization 70.470 Partial Differential Equations I 70.471 Partial Differential Equations II
- 70.472 Integral Transforms
- 70.473 Qualitative Theory of Ordinary **Differential Equations**
- 70.482 Introduction to Mathematical Logic
- 70.483 Topics in Applied Logic
- 70.484 Design and Analysis of Algorithms
- 70.485 Theory of Automata
- 70.486 Numerical Analysis
- 70.487 Game Theory
- 70.488 Graph Theory and Algorithms

Graduate Courses*

• Mathematics 70.501 W1 (MAT5120) Abstract Measure Theory Abstract measure and integral, L-spaces, complex measures, product measures, differentiation theory, Fourier transforms.

Prerequisite: Mathematics 70.407.

• Mathematics 70.502F1 (MAT5123)

Distributions and Generalized Functions Linear topological spaces, countably multinormed spaces, countable union spaces and their duals, testing function spaces, spaces of generalized functions and their structure, Schwartz distributions, calculus of distribution, convolution, analytic representation, and Fourier transform of distributions. Prerequisite: Mathematics 70.403.

 Mathematics 70.503F1 (MAT5122) Banach Algebras

Commutative Banach algebras; the space of maximal deals; representation of Banach algebras as function algebras and as operator algebras; the spectrum of an element. Special types of Banach algebras; for example, regular algebras with involution, applicaions.

Mathematics 70.504F1 (MAT5129) Integral Equations

A survey of the main results in the theory of nonsingular linear integral equations; Volterra and Fredholm equations of first and second kind in the L_2 case, with special results for the continuous case; Hermitian kernels; eigen-function expansions; compact operators.

Prerequisites: Mathematics 70.302 and 70.403.

F,W,S indicates term of offering. Courses offered in the fall and vinter will be followed by T.

The number following the letter indicates the credit weight of the course: 1 denotes a half-course credit, 2 denotes a full-course credit,

• Mathematics 70.505F1 (MAT5127)

Complex Analysis

Complex differentiation and integration, harmonic functions, maximum modulus principle, Runge's theorem, conformal mapping, entire and meromorphic functions, analytic continuation.

• Mathematics 70.506F1 (MAT5316)

Topological Vector Spaces

Construction of new topological vector spaces out of given ones; local convexity and the Hahn-Banach theorem; compactness and the Krein-Milman theorem; conjugate spaces, polar sets. *Prerequisite:* Mathematics 70.403.

• Mathematics 70.507F1 (MAT5125)

Real Analysis I (Measure Theory and Integration) General measure and integral, Lebesgue measure and integration on R, Fubini's theorem, Lebesgue-Radon-Nikodym theorem, absolute continuity and differentiation, LP-spaces. Selected topics such as Daniell-Stone theory.

Prerequisites: Mathematics 70.301 and 70.302 (MAT3125) or permission of the Department.

• Mathematics 70.508W1 (MAT5126)

Real Analysis II (Functional Analysis) Banach and Hilbert spaces, bounded linear operators, dual spaces. Topics selected from: weak- and weak-topologies, Alaoglu's theorem, compact operators, differential calculus in Banach spaces, Riesz representation theorems.

Prerequisite: Mathematics 70.507 (MAT5125) or permission of the Department.

• Mathematics 70.509F1 (MAT5121) Introduction to Hilbert Space Geometry of Hilbert Space, spectral theory of linear operators in Hilbert Space.

Prerequisites: Mathematics 70.301, 70.302, and 70.403.

• Mathematics 70.512F1 (MAT5148) Group Representations and Applications An introduction to group representations and character theory, with selected applications.

• Mathematics 70.513F1 (MAT5146) Rings and Modules

Generalizations of the Wedderburn-Artin theorem and applications, homological algebra.

• Mathematics 70.514F1 (MAT5143) Lie Algebras

Basic concepts; ideals, homomorphisms, nilpotent, solvable, semi-simple. Representations, universal enveloping algebra. Semi-simple Lie algebras:

structure theory, classification, representation theory.

Prerequisites: Mathematics 70.517 (MAT5141) and 70.519 (MAT5142) or permission of the Department.

• Mathematics 70.516W1 (MAT5145) Group Theory

Fundamental principles as applied to abelian, nilpotent, solvable, free, and finite groups; representations. *Prerequisite:* Mathematics 70.310 or permission of the Department.

• Mathematics 70.517F1 (MAT5141) Algebra I

Groups, Sylow subgroups, finitely generated abelian groups. Rings, field of fractions, principal ideal domains, modules. Polynomial algebra, Euclidean algorithm, unique factorization.

Prerequisite: Permission of the Department.

• Mathematics 70.518W1 (MAT5147)

Homological Algebra and Category Theory Axioms of set theory, categories, functors, natural transformations; free, projective, injective and flat modules; tensor products and homology functors, derived functors; dimension theory.

Prerequisite: Mathematics 70.310 or permission of the Department.

• Mathematics 70.519W1 (MAT5142) Algebra II

Field theory, algebraic and transcendental extensions, finite fields, Galois groups. Modules over principal ideal domains, decomposition of a linear transformation, Jordan normal form.

Prerequisites: Mathematics 70.517 (MAT5141) and permission of the Department.

• Mathematics 70.521W1 (MAT5150)

Topics in Geometry

Various axiom systems of geometry. Detailed examinations of at least one modern approach to foundations, with emphasis upon the connections with group theory.

Prerequisite: Permission of the Department.

• Mathematics 70.522F1 (MAT5168)

Homology Theory

The Eilenberg-Steenrod axioms and their consequences, singular homology theory, applications to topology and algebra. *Prerequisite:* Mathematics 70.425.

Prerequisite: Mathematics 70.425.

• Mathematics 70.525F1 (MAT5151) Topology I

Topological spaces, product and identification topologies, countability and separation axioms,

compactness, connectedness, metrization, net and filter convergence.

Prerequisite: Mathematics 70.301 or permission of the Department.

• Mathematics 70.526W1 (MAT5152)

Topology II

Homotopy, fundamental group, covering spaces, complexes, classification of two-dimensional manifolds.

Prerequisites: Mathematics 70.310 (MAT3143) and 70.525 (MAT5151) or permission of the Department.

• Mathematics 70.527F1 (MAT5169)

Foundations of Geometry

A study of at least one modern axiom system of Euclidean and non-Euclidean geometry, embedding of hyperbolic and Euclidean geometries in the projective plane, groups of motions, models of non-Euclidean geometry.

Prerequisite: Mathematics 70.310 (may be taken concurrently) or permission of the Department.

• Mathematics 70.528F1 (MAT5155) Differentiable Manifolds

A study of differentiable manifolds from the point of view of either differential topology or differential geometry. Topics such as smooth mappings, transversality, intersection theory, vector fields on manifolds, Gaussian curvature, Riemannian manifolds, differential forms, tensors, and connections are included.

Prerequisite: Mathematics 70.301 or permission of the Department.

• Mathematics 70.531F1 (MAT5161) Mathematical Logic

A basic graduate course in mathematical logic. Propositional and predicate logic, proof theory, Gentzen's Cut-Elimination, completeness, compactness, Henkin models, model theory, arithmetic and undecidability. Special topics (time permitting) depending on interests of instructor and audience. Honours undergraduate alegebra, analysis and topology or permission of the instructor.

• Mathematics 70.535F1 (MAT5163) Analytic Number Theory

Dirichlet series, characters, Zeta-functions, prime number theorem, Dirichlet's theorem on primes in arithmetic progressions, binary quadratic forms. *Prerequisite:* Mathematics 70.307 or permission of the Department. • Mathematics 70.536W1 (MAT5164)

Algebraic Number Theory

Algebraic number fields, bases, algebraic integers, integral bases, arithmetic in algebraic number fields, ideal theory, class number. *Prerequisite:* Mathematics 70.310 or permission of the Department.

• Mathematics 70.540F1 (MAT5185) Advanced Classical Mechanics Hamiltonian dynamics, integral invariants, nonholonomic systems, rigid body motions. *Prerequisite:* Mathematics 70.345 or permission of the Department.

• Mathematics 70.541F1 (MAT5320)

Calculus of Variations

Extreme values of functionals; necessary conditions for an extremum. Sufficient conditions for an extremum. Hamilton-Jacobi Theory and the Maximum Principle of Pontryagin. The problem of Lagrange; the Isoperimetric problem.

Prerequisite: Mathematics 70.345 or permission of the Department.

• Mathematics 70.542W1 (MAT5186) Special Functions

Hypergeometric and Generalized Hypergeometric functions; classical orthogonal polynomials in discrete and continuous variables. Confluent, Hypergeometric and Bessel functions. Asymptotic expansions; steepest descent, WKBJ approximation and other asymptotic methods.

Prerequisites: Mathematics 70.307 and 70.308, or permission of the Department.

• Mathematics 70.545F1 (MAT5131)

Ordinary Differential Equations

Existence and uniqueness theorems, boundary value problems, qualitative theory.

Prerequisite: Mathematics 70.308 or permission of the Department.

• Mathematics 70.546F1 (MAT5133) Introduction to Partial Differential Equations First order linear, quasi-linear, and nonlinear equations; second order equations in two or more variables; systems of equations; the wave equation; Laplace and Poisson equations; Dirichlet and Neumann problems; Green's functions. *Prerequisites:* Mathematics 70.302, or 70.307 and 70.308, or permission of the Department.

• Mathematics 70.547W1 (MAT5134) Topics in Partial Differential Equations Theory of distributions, initial-value problems based on two-dimensional wave equations, Laplace transform, Fourier integral transform, diffusion problems, Helmholtz equation with application to boundary and initial-value problems in cylindrical and spherical coordinates.

Prerequisite: Mathematics 70.546 or permission of the Department.

• Mathematics 70.550F1 (MAT5177)

Multivariate Normal Theory

Multivariate normal distribution properties, characterization, estimation of means, and covariance matrix. Regression approach to distribution theory of statistics; multivariate tests; correlations; classification of observations; Wilks' criteria. *Prerequisite:* Mathematics 70.350.

• Mathematics 70.551W1 (MAT5191) Mathematical Statistics II

Confidence intervals and pivotals; Bayesian intervals; optimal tests and Neyman-Pearson theory; likelihood ratio and score tests; significance tests; goodness-of-fit-tests; large sample theory and applications to maximum likelihood and robust estimation. *Prerequisite:* Mathematics 70.450 or 70.560 or permission of the Department.

• Mathematics 70.552W1 (MAT5192) Sampling Theory and Methods

Unequal probability sampling with and without replacement; unified theory for standard errors; prediction approach; ratio and regression estimation; stratification and optimal designs; multistage cluster sampling; double sampling; domains of study; post-stratification; nonresponse; measurement errors; related topics.

Prerequisite: Mathematics 70.452 or permission of the Department.

• Mathematics 70.553F1 (MAT5193)

Linear Models

Theory of non full rank linear models; estimable functions, best linear unbiased estimators, hypotheses testing, confidence regions; multi-way classifications; analysis of covariance; variance component models; maximum likelihood estimation, Minque, Anova methods; miscellaneous topics.

Prerequisite: Mathematics 70.450 or permission of the Department.

• Mathematics 70.554F1 (MAT5194)

Stochastic Processes and Time Series Analysis Stationary Stochastic processes, inference for stochastic processes, applications to time series and spatial series analysis.

Prerequisite: Mathematics 70.451 or permission of the Department.

• Mathematics 70.555W1 (MAT5195) Design of Experiments

Overview of linear model theory; orthogonality; randomized block and split plot designs; latin square designs; randomization theory; incomplete block designs; factorial experiments: confounding and fractional replication; response surface methodology. Miscellaneous topics.

Prerequisite: Mathematics 70.355 or 70.450 or permission of the Department.

• Mathematics 70.556W1 (MAT5175)

Robust Statistical Inference

Nonparametric tests for location, scale, and regression parameters; derivation of rank tests; distribution theory of linear rank statistics and their efficiency. Robust estimation of location, scale and regression parameters; Huber's M-estimators, Rank-methods, L-estimators. Influence function. Adaptive procedures. *Prerequisite:* Mathematics 70.450 or permission of the Department.

• Mathematics 70.557W1 (MAT5176)

Advanced Statistical Inference

Pure significance test; uniformly most powerful unbiased and invariant tests; asymptotic comparison of tests; confidence intervals; large-sample theory of likelihood ratio and chi-square tests; likelihood inference; Bayesian inference and topics such as empirical Bayes inference; fiducial and structural methods; resampling methods.

Prerequisite: Mathematics 70.457 or 70.551 or permission of the Department.

• Mathematics 70.558F1 (MAT5172)

Topics in Stochastic Processes

Course contents will vary, but will include topics drawn from Markov processes. Brownian motion, stochastic differential equations, martingales, Markov random fields, random measures, and infinite particle systems, advanced topics in modelling, population models, etc.

Prerequisites: Mathematics 70.356 or 70.451, or permission of the Department.

• Mathematics 70.559F1 (MAT5196)

Multivariate Analysis

Multivariate methods of data analysis, including principal components, cluster analysis, factor analysis, canonical correlation, MANOVA, profile analysis, discriminant analysis, path analysis.

Prerequisite: Mathematics 70.450 or permission of the Department.

• Mathematics 70.560F1(MAT5190)

Mathematical Statistics I

Statistical decision theory; likelihood functions; sufficiency; factorization theorem; exponential families; UMVU estimators; Fisher's information; Cramer-Rao lower bound; maximum likelihood and moment estimation; invariant and robust point estimation; asymptotic properties; Bayesian point estimation.

Prerequisite: Mathematics 70.350 or permission of the Department.

• Mathematics 70.561F1 (MAT5197)

Stochastic Optimization

Topics chosen from stochastic dynamic programing, Markov decision processes, search theory, sequential inference problems, optimal stopping, analysis and solution of deterministic and stochastic modelling problems in the physical, social and life sciences. Students will present a paper on applications of particular interest to them.

Prerequisite: Mathematics 70.356 or permission of the Department.

• Mathematics 70.562F1 (MAT5317)

Analysis of Categorical Data

Analysis of one-way and two-way tables of nominal data; multi-dimensional contingency tables and log-linear models; tests of symmetry and marginal homogeneity in square tables; incomplete tables; tables with ordered categories; fixed margins and logistic models with binary response; measures of association and agreement; applications in biological, social and medical sciences.

Prerequisites: Mathematics 70.450, 70.457/70.551 or permission of the Department.

• Mathematics 70.563W1 (MAT5318)

Reliability and Survival Analysis

Types of censored data; nonparametric estimation of survival function; graphical procedures for model identification; parametric models and maximum likelihood estimation; exponential and Weibull regression models; nonparametric hazard function models and associate statistical inference; rank tests with censored data; engineering, medical and biological sciences applications.

Prerequisites: Mathematics 70.450, 70.457/70.551 or permission of the Department.

• Mathematics 70.565F1 (MAT5165) Theory of Automata

Algebraic structure of sequential machines, decomposition of machines; finite automata, formal languages; complexity.

Prerequisite: Mathematics 70.210 or permission of the Department.

• Mathematics 70.567F1 (MAT5324)

Game Theory

Two-person zero-sum games; infinite games; multistage games; differential games; utility theory; twoperson general-sum games; bargaining problem; n-person games; games with a continuum of players. *Prerequisite:* Mathematics 70.301 or permission of the Department.

• Mathematics 70.569F1 (MAT5301) Topics in Combinatorial Mathematics *Prerequisite:* Permission of the Department.

• Mathematics 70.571W1 (MAT5198) Stochastic Models

Markov systems, stochastic networks, queuing networks, spatial processes, approximation methods in stochastic processes and queuing theory. Applications to the modelling and analysis of computercommunications systems and other distributed networks.

Prerequisite: Mathematics 70.356 or permission of the Department.

• Mathematics 70.578F1 (MAT5170) Probability Theory I

Probability spaces, random variables, expected values as integrals, joint distributions, independence and product measures, cumulative distribution functions and extensions of probability measures, Borel-Cantelli lemmas, convergence concepts, independent identically distributed sequences of random variables. *Prerequisites:* Mathematics 70.301, 70.302 and 70.350 or permission of the Department.

• Mathematics 70.579W1 (MAT5171) Probability Theory II

Laws of large numbers, characteristic functions, central limit theorem, conditional probabilities and expectations, basic properties and convergence theorems for martingales, introduction to Brownian motion.

Prerequisite: Mathematics 70.578 (MAT5170) or permission of the Department.

• Mathematics 70.581F1 (MAT5303) Linear Optimization

Linear programing problems; simplex method, upper bounded variables, free variables; duality; postoptimality analysis; linear programs having special structures; integer programing problems; unimodularity; knapsack problem.

Prerequisite: Course in linear algebra and permission of the Department.

• Mathematics 70.582F1 (MAT5325)

Introduction to Information and Systems Science An introduction to the process of applying computers in problem-solving. Emphasis is placed on the design and analysis of efficient computer algorithms for large, complex problems. Applications in a number of areas are presented: data manipulation, databases, computer networks, queuing systems, optimization.

(Also offered as Engineering 94.582, Computer Science 95.582 and Information and Systems Science 93.582)

• Mathematics 70.583W1 (MAT5304) Nonlinear Optimization

Methods for unconstrained and constrained optimization problems; Kuhn-Tucker conditions; penalty functions; duality; quadratic programing; geometric programing; separable programing; integer nonlinear programing; pseudo-Boolean programing; dynamic programing.

Prerequisite: Permission of the Department.

• Mathematics 70.584F1, W1, S1 (MAT5307) Topics in Operations Research

• Mathematics 70.585F1, W1, S1 (MAT5308) Topics in Algorithm Design

• Mathematics 70.586F1 (MAT5180)

Numerical Analysis

Error analysis for fixed and floating point arithmetic; systems of linear equations; eigen-value problems; sparse matrices; interpolation and approximation, including Fourier approximation; numerical solution of ordinary and partial differential equations. *Prerequisite:* Permission of the Department.

• Mathematics 70/95.587F1 (MAT5167) Formal Language and Syntax Analysis Computability, unsolvable and NP-hard problems. Formal languages, classes of language automata. Principles of compiler design, syntax analysis, parsing (top-down, bottom-up), ambiguity, operator precedence, automatic construction of efficient parsers, LR, LR(O), LR(k), SLR, LL(k). Syntax directed translation.

Prerequisites: Mathematics 70.565 or 70.485 or Computer Science 95.302, or permission of the Department.

• Mathematics 70.588W1 (MAT5305) Combinatorial Optimization

Network flow theory and related material. Topics will include shortest paths, minimum spanning trees, maximum flows, minimum cost flows. Optimal matching in bipartite graphs.

Prerequisite: Permission of the Department.

• Mathematics 70.589W1 (MAT5306) Combinatorial Optimization

Topics include optimal matching in non-bipartite graphs, Euler tours and the Chinese Postman problem. Other extensions of network flows: dynamic flows, multicommodity flows, and flows with gains, Bottleneck problems. Matroid optimization. Enumerative and heuristic algorithms for the Travelling Salesman and other "hard" problems. *Prerequisite:* Mathematics 70.588.

• Mathematics 70.590F1, W1, S1 (MAT5990) Seminar

• Mathematics 70.591F1, W1, S1 (MAT5991) Directed Studies

• Mathematics 70.593F1, W1, S1

Project

This course is intended for students registered in the M.Sc. degree program in Information and Systems Science and the M.C.S. program. Students pursuing the non-thesis option will conduct a study, analysis, and/or design project under the supervision of a faculty member. Results will be given in the form of a type-written report and presented at a departmental seminar.

• Mathematics 70.594F1, W1, S1 Statistical Internship

This course is project-oriented and affords students the opportunity to undertake statistical research and data analysis projects either within the Statistical Consulting Centre or as a cooperative project with governmental or industrial sponsors. In addition to project work, seminars on related topics will be conducted. Practical data analysis and consulting skills will be emphasized. The grade assigned in this course will be based upon oral and written presentation of analysis results and will be determined in consultation with the faculty adviser and the sponsor. Permission of the Institute is required for registration in this course.

• Mathematics 70/94/95.595F4, W4, S4 M.C.S. Thesis

• Mathematics 70/93/94/95.598 F3, W3, S3 M.Sc. Thesis in Information and Systems Science

• Mathematics 70.599F2, W2, S2 M.Sc. Thesis

• Mathematics 70.602W1 (MAT5309) Harmonic Analysis on Groups Transformation groups; Haar measure; unitary representations of locally compact groups; completeness and compact groups; character theory; decomposition.

• Mathematics 70.608F1, W1, S1 (MAT5326) Topics in Analysis

• Mathematics 70.609F1, W1, S1 (MAT5329) Topics in Analysis

• Mathematics 70.611F1, W1, S1 (MAT5327) Topics in Algebra • Mathematics 70.612F1, W1, S1 (MAT5330) Topics in Algebra

• Mathematics 70.613F1, W1, S1 (MAT5331) Topics in Algebra

• Mathematics 70.614W1 (MAT5158) Lie Groups

Matrix groups: one-parameter groups, exponential map, Campbell-Hausdorff formula, Lie algebra of a matrix group, integration on matrix groups. Abstract Lie groups.

Prerequisites: Mathematics 70.507 and 50.517 or permission of the Department.

• Mathematics 70.621F1, W1, S1 (MAT5312) Topics in Topology

• Mathematics 70.657F1, W1, S1 (MAT5313) Topics in Probability and Statistics

• Mathematics 70.658F1, W1, S1 (MAT5314) Topics in Probability and Statistics

• Mathematics 70.686F1, W1, S1 (MAT5361) Topics in Mathematical Logic

• Mathematics 70.687F1 (MAT5162) Mathematical Foundations of Computer Science Foundations of functional languages, lambda calculi (typed, polymorphically typed, untyped), Curry-Howard Isomorphism, proofs-as-programs, normalization and rewriting theory, operational semantics, type assignment, introduction to denotational semantics of programs, fixed-point programing. Topics chosen from: denotational semantics for lambda calculi, models of programing languages, complexity theory and logic of computation, models of concurrent and distrubted systems, etc.

Prerequisites: Honours undergraduate algebra and either topology or analysis. Some acquaintance with logic useful, or permission of the Instructor.

• Mathematics 70.690F1, W1, S1 (MAT6990) Seminar

• Mathematics 70.691F1, W1, S1 (MAT6991) Directed Studies

• Mathematics 70.699F, W, S Ph.D. Thesis